# MARIA COLLEGE OF ENGINEERING AND TECHNOLOGY, ATTOOR.

## B.E.Degree Examination
## (common to all branches)

## TRANSFORMS AND PARTIAL DIFGFERENTIAL EQUATION (MA31)

### 2 MARK QUESTIONS AND ANSWERS

## UNIT 1

## FOURIER SERIES

**1. Find $b_n$ in the expansion of $x^2$ as a Fourier series in $(-\pi, \pi)$?**

Solution:

Given $f(x) = x^2$ is an even function.

[$\because f(x) = x^2$ and $f(-x) = x^2$

$\therefore f(x) = f(-x)$

$\therefore f(x)$ is an even function ]

$\therefore b_n = 0$

**2. If $f(x)$ is an odd function defined in $(-l, l)$ what are the values of $a_0$ and $a_n$ ?**

Solution:

Given $f(x)$ is an odd function in $(-l, l)$

$\therefore a_0 = 0 \qquad a_n = 0$

**3. Find the Fourier constants $b_n$ for $xsinx$ in$(-\pi, \pi)$?**

Solution:

Let $f(x) = xsinx$

$f(-x) = -xsin(-x)$

$= -x(-sinx)$

$= xsinx$

$\therefore f(x) = f(-x)$

$\therefore f(x) = xsinx$ is an even function

$\therefore b_n = 0$

**4. State the Parseval's identity for the half-range cosine expansion of $f(x)$ in $(0,1)$ ?**

Solution:

Parseval's identity for the half-range cosine expansion of $f(x)$ in $(0,1)$ is

$$\frac{1}{b-a}\int_a^b [f(x)]^2 dx = \left(\frac{a_0}{2}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty} a_n{}^2$$

**5. Find the value of $a_n$ in the cosine series expansion of $f(x) = k$ in the interval $(0,10)$?**

Solution:

Given $f(x) = k$ in $(0,10)$.Here $l = 10$

$$a_n = \frac{2}{l}\int_0^l f(x)cos\frac{n\pi x}{l}dx$$

$$= \frac{2}{10}\int_0^{10} kcos\frac{n\pi x}{10}dx$$

$$= \frac{k}{5}\left[\frac{sin\frac{n\pi x}{10}}{\frac{n\pi}{10}}\right]_0^{10}$$

$$= \frac{k}{5}\frac{10}{n\pi}sin\frac{10n\pi}{10}$$

$$= \frac{2k}{n\pi}sinn\pi$$

=0.

**6. Determine $b_n$ in the Fourier series expansion of $f(x) = \frac{1}{2}(\pi - x)$ in $0 < x < 2\pi$ with period ?**

Solution:

Given $f(x) = \frac{1}{2}(\pi - x)$ in $0 < x < 2\pi$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sin\,nxdx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\frac{1}{2}(\pi - x)sin\,nxdx$$

$$= \frac{1}{2\pi}\int_0^{2\pi}(\pi - x)sin\,nxdx$$

$$= \frac{1}{2\pi}\left[(\pi - x)\left(\frac{-cosnx}{n}\right) - \frac{sinnx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[(\pi - 2\pi)\left(\frac{-cos2n\pi}{n}\right) - \frac{sin2n\pi}{n^2} - \left|(\pi - 0)\left(\frac{-cos0}{n}\right) - \frac{sin0}{n^2}\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{n} + \frac{\pi}{n}\right]$$

$$= \frac{2\pi}{2n\pi}$$

$$= \frac{1}{n}$$

**7. Define Root Mean Square value of a function $f(x)$ in $a < x < b$?**

Solution:

Root Mean Square value= $\sqrt{\frac{\int_a^b [f(x)]^2 dx}{b-a}}$ in the interval $a < x < b$.

**8. Determine the value of $a_n$ in the Fourier Series expansion of $f(x) = x^3$ in $-\pi < x < \pi$?**

Solution:

Given $f(x) = x^3$

$f(-x) = (-x)^3$

$f(-x) = -f(x)$

$\because f(x) = x^3$ is an odd function.

$\therefore a_n = 0$

**9. The Fourier series expansion of $f(x)$ in $(0, 2\pi)$ is $f(x) = \sum_{n=1}^{\infty} \frac{sinnx}{n}$.Find the Root mean square value of $f(x)$in the interval$(0, 2\pi)$?**

Solution:

Given $f(x) = \sum_{n=1}^{\infty} \frac{sinnx}{n}$

$\therefore a_0 = 0 \quad a_n = 0 \quad b_n = \frac{1}{n}$

Root mean square value of $f(x)$in the interval $(0,2\pi)$ is

$$\bar{y}^2 = \left(\frac{a_0}{2}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left({a_n}^2 + {b_n}^2\right)$$

$$= 0 + \frac{1}{2}\sum_{n=1}^{\infty}\left(0 + \frac{1}{n^2}\right)$$

$= \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$

**10. State the sufficient condition for a function $f(x)$ to be expressed as Fourier series (or) Explain Dirichlet's conditions?**

Solution:

(i) $f(x)$ is periodic ,single valued and finite

(ii) $f(x)$ has a finite number of finite discontinuities in any one period and no infinite discontinuity .

(iii) $f(x)$ has at the most a finite number of maxima and minima.

**11. Obtain the sum of the Fourier series for $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 & 1 < x < 2 \end{cases}$ at $x = 1$**

Solution:

$x = 1$ is a point of discontinuity

Sum$= \frac{f(1-)+f(1+)}{2}$

$= \frac{1+2}{2}$

$= \frac{3}{2}$

**12. If $f(x) = 2x$ in the interval $(0, 4)$, then find the value of $a_2$ in the Fourier series expansion ?**

Solution:

Given $f(x) = 2x$. Here $2l = 4$; $l = 2$

We know that $a_n = \frac{2}{l}\int_0^l f(x)cos\frac{n\pi x}{l}dx$

$a_n = \frac{1}{2}\int_0^4 2xcos\frac{n\pi x}{2}dx$

$= \frac{1}{2}\int_0^4 2xcos\pi x dx$

$= \left|x\left(\frac{sin\pi x}{\pi}\right) - (1)\left(\frac{-cos\pi x}{\pi^2}\right)\right]_0^4$

$= \left|x\left(\frac{sin\pi x}{\pi}\right) + \frac{cos\pi x}{\pi^2}\right]_0^4$

$= \left(0 + \frac{1}{\pi^2}\right) - \left(0 + \frac{1}{\pi^2}\right)$

$= 0.$

**13. Find the Sine series of $f(x) = k$ in $(0, \pi)$ ?**

Solution:

Given $f(x) = k$ in $(0, \pi)$

Sine series is $f(x) = \sum_{n=1}^{\infty} b_n sinnx$

Where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) sin\, nxdx$

$$= \frac{2}{\pi}\int_0^{\pi} ksin\, nxdx$$

$$= \frac{2k}{\pi}\int_0^{\pi} sin\, nxdx$$

$$= \frac{2k}{\pi}\left[-\frac{cosnx}{n}\right]_0^{\pi}$$

$$= \frac{2k}{\pi}\left|-\frac{cosn\pi}{n} - \left(\frac{cos0}{n}\right)\right]$$

$$= \frac{2k}{\pi}[-(-1)^n + 1]$$

$$= \frac{2k}{\pi}[1 - (-1)^n]$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\left[\frac{2k}{\pi}[1 - (-1)^n]\right| sinnx$$

$$= \frac{2k}{\pi}\sum_{n=1}^{\infty}\big[[1 - (-1)^n]\big]sinnx$$

$$= \frac{2k}{\pi}\left[2sinnx + \frac{2sin3x}{3} + \cdots\right|$$

$$= \frac{4k}{\pi}\left|sinnx + \frac{sin3x}{3} + \cdots\right|$$

**14. If $f(x) = sinhx$ is defined in $-\pi < x < \pi$ write the value of $a_0, a_n$?**

Solution:

Given $f(x) = sinhx$

$$f(-x) = \sinh(-x)$$

$$= -sinhx$$

$$= -f(x)$$

$$\therefore f(-x) = f(x)$$

$\therefore f(x)$ is an odd function .

$\therefore\ a_0 = 0\ , a_n = 0.$

**15. If $f(x) = x^2 + x$ is expressed as a Fourier Series in the interval $(-2, 2)$ to which value in this series converges at $x = 2$?**

Solution:

Given $f(x) = x^2 + x$

$x = 2$ is discontinuous.

$\therefore f(2) = \frac{f(-2)+f(2)}{2}$

$= \frac{4-2+4+2}{2}$

$= 4$

$\therefore$ The Fourier series converges at $x = 2$ to the value 4.

**16. If the Fourier series corresponding to $f(x) = x$ the interval il $(0, 2\pi)$ is**

$\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)$ **Without finding the value of $a_0, a_n, b_n$ find the value**

$\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}(a_n{}^2 + b_n{}^2)$**?**

Solution:

By Parseval's identity

$$\frac{a_0{}^2}{2} + \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx) = \frac{1}{2\pi-0}\int_0^{2\pi} x^2 dx$$

$$= \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\frac{(2\pi)^3}{3}$$

$$= \frac{4\pi^2}{3}$$

**17. What do you mean by Harmonic Analysis?**

Solution:

The process of finding the Fourier Series for a function $y = f(x)$from the tabulated values of $x$ and $y$ at equal intervals of $x$ is called Harmonic Analysis.

**18. Does $f(x) = tanx$ posses a Fourier expansion?**

Solution:

$f(x) = tanx$ has an infinite discontinuity Dirichlet's condition is not satisfied .Hence Fourier expansion does not exite.

**19. If the Fourier Series for the function $f(x) = \begin{cases} 0 & 0 < x < \pi \\ sinx & \pi < x < 2\pi \end{cases}$ is**

**$f(x) = -\frac{1}{\pi} + \frac{2}{\pi}\left[\frac{cos2x}{1.3} + \frac{cos4x}{3.5} + \frac{cos6x}{5.7} + \cdots\right| + \frac{sinx}{2}$ deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi-2}{4}$?**

Solution:

Given $f(x) = \begin{cases} 0 & 0 < x < \pi \\ sinx & \pi < x < 2\pi \end{cases}$

$x = \frac{\pi}{2}$ is continuous

$$f\left(\frac{\pi}{2}\right) = -\frac{1}{\pi} + \frac{2}{\pi}\left[\frac{cos2\frac{\pi}{2}}{1.3} + \frac{cos4\frac{\pi}{2}}{3.5} + \frac{cos6\frac{\pi}{2}}{5.7} + \cdots\right| + \frac{sin\frac{\pi}{2}}{2}$$

$$f\left(\frac{\pi}{2}\right) = -\frac{1}{\pi} + \frac{2}{\pi}\left[\frac{cos\pi}{1.3} + \frac{cos2\pi}{3.5} + \frac{cos3\pi}{5.7} + \cdots\right| + \frac{sin\frac{\pi}{2}}{2}$$

$$0 = -\frac{1}{\pi} + \frac{2}{\pi}\left[-\frac{1}{1.3} + \frac{1}{3.5} - \frac{1}{5.7} + \cdots\right| + \frac{1}{2}$$

$$\frac{1}{\pi} - \frac{1}{2} = -\frac{2}{\pi}\left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right|$$

$$\frac{2-\pi}{2\pi} = -\frac{2}{\pi}\left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right|$$

$$\frac{2-\pi}{2\pi}\left(\frac{-\pi}{2}\right) = \left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right|$$

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi-2}{4}$$

**20. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$ is**

**$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{n^2}cosnx - \frac{2}{n}sinnx\right)$ then find the value of infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \cdots$?**

Solution:

Given $f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{n^2}cosnx - \frac{2}{n}sinnx\right)$

$x = \pi$ is discontinuous

$$\frac{f(\pi)+f(-\pi)}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{n^2}cosn\pi - \frac{2}{n}sinn\pi\right)$$

$$\frac{\pi+\pi^2-\pi+(-\pi)^2}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\frac{2\pi^2}{2} = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\pi^2 - \frac{\pi^2}{3} = 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\frac{3\pi^2-\pi^2}{3} = 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \cdots\right)$$

$$\frac{2\pi^2}{3.4} = \left(\frac{1}{1^2} + \frac{1}{2^2} + \cdots\right)$$

$$\frac{\pi^2}{6} = \left(\frac{1}{1^2} + \frac{1}{2^2} + \cdots\right)$$

**PART B**

1.Obtain the Fourier series of $f(x) = x + x^2$ in $(-\pi,\pi)$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

2.If $f(x) = \begin{cases} 1-x & -\pi \leq x \leq 0 \\ 1+x & 0 \leq x \leq \pi \end{cases}$ Find the Fourier series for $f(x)$ and hence deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

3. Obtain the Fourier series upto the second Harmonic from the data

| $t$ | 0 | $\frac{T}{6}$ | $\frac{T}{3}$ | $\frac{T}{2}$ | $\frac{2T}{6}$ | $\frac{5T}{6}$ | $T$ |
|---|---|---|---|---|---|---|---|
| $y$ | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

4.Compute the first Harmonic of the Fourier series of $f(x)$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 4 | 8 | 15 | 7 | 6 | 2 |

5.Show that the complex form of the Fourier series of the periodic function $f(x) = e^{-x}, -1 < x < 1$ and $f(x+2) = f(x)$ is $f(x) = \sum_{-\infty}^{\infty}\frac{(-1)^n(1-in\pi)}{1+n^2\pi^2}sinh1(e^{in\pi x})$

6.Express $f(x) = x$ in cosine series in the range $0 < x < L$ and deduce that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$

7.Find the Half range sine series for a function $f(x) = x(\pi - x) \quad 0 < x < \pi$.Hence deduce that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3} = \frac{\pi^3}{32}$

8.Find the Fourier series for the function $f(x) = |x|, -l < x < l$. Hence find the value of

$1^{-2} + 3^{-2} + 5^{-2} + \cdots$

## UNIT: II

## FOURIER TRANSFORM

### UNIT: 2

### FOURIER TRANSFORM

1. **Find the Fourier transform of $f(x) = \begin{cases} e^{ikx}, a < x < b \\ 0, x \le a \text{ and } x > b \end{cases}$**

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x).e^{isx}\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_a^b e^{ikx}.e^{isx}dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_a^b e^{i(k+s)x}dx\right|$$

$$= \frac{1}{\sqrt{2\pi}} \left[\left[\frac{e^{i(k+s)x}}{i(k+s)}\right]_a^{\ b}\right]$$

$$= \frac{1}{\sqrt{2\pi}i(k+s)} \left[e^{i(k+s)b} - e^{i(k+s)a}\right]$$

**2. Find the Fourier sine transform of $\frac{1}{x}$**

We know that

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) sinsxdx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{1}{x} sinsxdx$$

$$= \sqrt{\frac{2}{\pi}}.\frac{\pi}{2}$$

$$= \sqrt{\frac{\pi}{2}}.$$

**3. State the Fourier integral theorem.**

The Fourier integral theorem of f(x) in the interval $(-\ell,\ell)$ is

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t)\cos\lambda(t-x)\,dxd\lambda$$

**4. Write down the Fourier cosine transform pair of formulae.**

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) cossxdx$$

$$F^{-1}[F_c[f(x)]] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F_c[f(x)] cossxds$$

$F_c[f(x)]$ and $F^{-1}[F_c[f(x)]]$ are called Fourier cosine transform pair.

**5. Define Fourier Transform pair.**

(i) F[f(x)]= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x).\,e^{isx}\,dx$

(ii)Inversion formulae

f(x)= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F[f(x)].\,e^{-isx}\,dx$

**6. Define Fourier Transform sine transform pair formulae.**

(i) Fourier sine Transform

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x) sinsxdx$$

(ii)Inversion Formulae

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F_s[f(x)] sinsxdx.$$

**7. State the Fourier transform of the derivatives of a function**.

The Fourier transform of $F^1(x)$,the derivative of F(x) is f(s), where f(s) is the Fourier transform of F(x).

$$F[F^1(x)] = is\,f(s)\ .$$

**8. State the Convolution Theorem for Fourier transforms.**

The Fourier transform of the convolution of f(x) and g(x) is the product of their Fourier transforms.

(i.e)F[f(x)*g(x)]=F(s)G(s)=F[f(x)]F[(g(x)]

**9. State the Parseval's identity on Fourier transform.**

If f(x) is a given function defined in (-∞, ∞) then it satisfy the identity

$\int_{-\infty}^{\infty}|f(x)|^2dx = \int_{-\infty}^{\infty}|F(s)|^2ds$.where F(s) is the Fourier Transform of f(x).

**10. State the modulation theorem in Fourier Transform.**

If F(s) is the Fourier Transform of f(x), then F[f(x) cosax]=$\frac{1}{2}$[F(s+a)+F(s-a)].

**11. What is the Fourier Transform of f(x-a) if the Fourier transform of f(x) is f(s).**

F[f(x-a)]=$e^{ias}f(s)$ .

**12. Find the Fourier sine transform of f(x)=$e^{-x}$.**

We know that $F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)sinsxdx$

$$F_s[e^{-x}] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-x}sinsxdx$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{s}{1+s^2}\right] \qquad [\because \int_0^{\infty} e^{-ax}sinbxdx = \frac{b}{a^2+b^2}]$$

**13. Prove that F [f(ax)]=$\frac{1}{a}F\left(\frac{s}{a}\right)$, $a > 0$.**

F[f(ax)]= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(ax).e^{isx}\,dx$

ax=y

a dx= dy (i.e), dx=$\frac{dy}{a}$

when x=-∞, $y = -\infty \; and \; x = \infty, y = \infty$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(y).e^{is\frac{y}{x}}\frac{dy}{a}$$

$$=\frac{1}{a}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(y).e^{i\left(\frac{s}{a}\right)y}.dy$$

$$=\frac{1}{a}F(\frac{s}{a}).$$

**14. If f(x)= $e^{-ax}$,a<0,find Fourier sine transform of f(x).**

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x) sinsxdx$$

$$F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-ax} sinsxdx \qquad [\because \int_0^\infty e^{-ax} sinbxdx = \frac{b}{a^2+b^2}]$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{s}{s^2+a^2}\right]$$

**15. State inverse theorem for complex Fourier transform.**

$$f(x)= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F[f(x)].e^{-isx}\,ds.$$

is called the inverse formula for the complex Fourier transform of F[f(x)].

**16. Let $F_c(s)$ be the Fourier cosine transform of f(x).Prove that**

$$\boldsymbol{F_c[f(x)cosax] = \frac{1}{2}[F_c(s+a) + F_c(s-a)]}.$$

$$F_c[f(x)cosax] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x) cosax\, cossxdx$$

$$= \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\{\cos(a+s)\,x + \cos(a-s)\,x\}dx$$

$$= \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(a+s)\,xdx + \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(a-s)\,xdx$$

$$= \frac{1}{2}[F_c(s+a) + F_c(s-a)]$$

**17. F [f(x)]=F(s) then prove that F$[\boldsymbol{e^{iax}f(x)}] = \boldsymbol{F(s+a)}$.**

$$F[e^{iax}f(x)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{iax}f(x).e^{isx}\,ds.$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x).e^{i(s+a)}\,dx$$

$$F\,[e^{iax}f(x)] = F(s+a)$$

**18. What is the sine transform of f (ax) if $\overline{f_s}$(s) is the Fourier sine transform of f(x).**

We know that

$$F_s[f(ax)] = \frac{1}{a}F_s(\frac{s}{a})$$

Where $F_s[f(x)] = F_s(s)$

Here it is given that $F_s[f(x)] = \overline{f_s}(s)$

$$F_s[f(ax)] = \frac{1}{a}\overline{f_s}\left(\frac{s}{a}\right).$$

**19. State Parseval's identity for the half-range cosine expansion of f(x) in (0, 1).**

$$2\int_0^1 [f(x)]^2\, dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

$$a_0 = 2\int_0^1 f(x)dx$$

$$a_n = 2\int_0^1 f(x) cosnx dx.$$

**20. Prove that $F_c$ [f (ax)] =$\frac{1}{a}F_c\left(\frac{s}{a}\right)$, $a > 0$.**

F[f(ax)]= $\sqrt{\frac{2}{\pi}}\int_0^\infty f(ax).cossx\, dx$

ax=y when x = 0,y=0

dx = $\frac{dy}{a}$ when x= $\infty, y = \infty$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty f(y) cos\left(\frac{sy}{a}\right).\frac{dy}{a}$$

$$=\frac{1}{a}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x).cos\left(\frac{s}{a}\right)x\, dx$$

$$=\frac{1}{a}F_c\left(\frac{s}{a}\right).$$

**PART B**

1.(i)Find the Fourier transform of $f(x) = \begin{cases} -1, & x < -1 \\ 1, & -1 \le x \le 1 \\ o, & x > 1 \end{cases}$

(ii)Find the Fourier transform of $f(x) = \begin{cases} |x| & for\ |x| < a \\ 0 & for\ |x| > a \end{cases}, a > 0$

2. Find the Fourier sine and cosine transform of $e^{-x}$ .Hence evaluate (i) $\int_0^\infty \frac{dx}{(x^2+1)^2}$ and (ii) $\int_0^\infty \frac{x^2 dx}{(x^2+1)^2}$

3. (i)Find the Fourier cosine transform of $e^{-x^2}$

(ii) Find the Fourier sine transform of $xe^{-x^2/2}$

4.Find the Fourier Transform of $f(x) = \begin{cases} 1-|x| & for\ |x| \le 1 \\ 0 & for\ |x| > 1 \end{cases}$ and also find the inverse transform . Hence deduce that $\int_0^\infty \left(\frac{sinx}{x}\right)^2 dx = \frac{\pi}{2}$ and $\int_0^\infty \left(\frac{sinx}{x}\right)^4 dx = \frac{\pi}{3}$

## UNIT:3

## PARTIAL DIFFERENTIAL EQUATIONS

**1.Form the partial differential equation by eliminating the arbitrary function from $z = f(xy)$?**

Given $z = f(xy)$

Differentiate partially with respect to $x$: $p = f'(xy)y \dots\dots (1)$

Differentiate partially with respect to $y$: $q = f'(xy)x \dots\dots (2)$

Eliminating $f$ from (1)& (2)

(1) □ $\frac{p}{y} = f'(xy)$

(2) □ $\frac{q}{x} = f'(xy)$

$\frac{p}{y} = \frac{q}{x}$

$px - qy = 0$

This is the required partial differential equation.

**2.Write down the complete solution of $z = px + qy + c\sqrt{1+p^2+q^2}$**

Given $z = px + qy + c\sqrt{1+p^2+q^2}$

The complete solution is $z = ax + by + c\sqrt{1+a^2+b^2}$

**3.Obtain partial differential equation by eliminate the arbitrary function from $z = f(x^2+y^2)$**

Given $z = f(x^2+y^2)$

Differentiate with respect to $x$: $p = f'(x^2+y^2)2x \dots\dots (1)$

Differentiate with respect to $y$: $q = f'(x^2+y^2)2y \dots\dots (2)$

Eliminating $f$ from (1)& (2)

(1) □ $\frac{p}{2x} = f'(x^2+y^2)$

(2) □ $\frac{q}{2y} = f'(x^2+y^2)$

□ $\frac{p}{2x} = \frac{q}{2y}$ □ $\frac{p}{x} = \frac{q}{y}$

$py - qx = 0$

This is the required partial differential equation.

**4.Find the complete integral of $p - y^2 = q + x^2$?**

Given $p - y^2 = q + x^2$

$p - x^2 = q + y^2 = k$

$\Rightarrow p - x^2 = k; \quad q + y^2 = k$

$\Rightarrow p = x^2 + k; \quad q = -y^2 + k$

By total derivative formula $dz = pdx + qdy$

$\Rightarrow dz = (x^2 + k)dx + (-y^2 + k)dy$

Integrating

$$z = kx + \frac{x^3}{3} + ky - \frac{y^3}{3} + C$$

5.Form the partial differential equation of all spheres whose centers lie on the $z$ −axis?

The equation of the sphere whose centers lie on the $z$ −axis is

$x^2 + x^2 + (z - c)^2 = r^2$ where $r$ is the constant .

Differentiate partially with respect to $x$

$2x + 2(z - c)p = 0 \ldots\ldots (1)$

Differentiate partially with respect to $x$

$2y + 2(z - c)q = 0 \ldots\ldots (2)$

From (2) $z - c = -\frac{y}{q}$

Substituting in (1) we get

$$x - \frac{y}{q}p = 0$$

$qx = py$

Which is the required partial differential equation.

**6.Form the p.d.e by eliminating the arbitrary constants from $z = ax + by + ab$**

Given:$z = ax + by + ab \ldots\ldots\ldots\ldots.. (1)$

$$P = \frac{\partial z}{\partial x} = a \ldots \ldots \ldots \ldots \ldots . . (2)$$

$$q = \frac{\partial z}{\partial y} = b \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

substituting (2)&(3) in (1) weget the required p.d.e

$$z = px + qy + pq$$

**7.Eliminate the arbitrary constants $a$&$b$ from $z = ax + by + a^2 + b^2$.**

Given: $z = ax + by + a^2+b^2$…………(1)

Differentiating (1) partially w.r.to'$x$' we get

$$\frac{\partial z}{\partial x} = a$$

$$P = a \ldots\ldots\ldots\ldots\ldots (2)$$

Differentiating(1) partially w.r.to 'y' we get

$$\frac{\partial z}{\partial y} = b$$

$$q = b \ldots \ldots \ldots \ldots \ldots . (3)$$

Substitute in equation (1) we get the required p.d.e

$$Z = px + qy + p^2 + q^2$$

**8. Form a p.d.e by eliminating the arbitrary constants a&b from z=$(x + a)^2 - (y - b)^2$**

Given z=$(x + a)^2 - (y - b)^2 \ldots \ldots \ldots \ldots . (1)$

Differentiating (1) partially w.r.to'$x$' we get

$$\frac{\partial z}{\partial x} = 2(x + a)$$

$$P = 2(x + a) \ldots\ldots\ldots\ldots\ldots (2)$$

Differentiating(1) partially w.r.to 'y' we get

$$\frac{\partial z}{\partial y} = 2(y - b)$$

$q = 2(y - b) \ldots \ldots \ldots \ldots \ldots .(3)$

Substitute in equation (1) we get the required p.d.e

$z = \left(\frac{p}{2}\right)^2 - \left(\frac{q}{2}\right)^2$

$\Rightarrow 4z = p^2 - q^2$

**9.Find the partial differential equation of all planes having equal intercepts on the $x$ and $y$ axis?**

The equation of such plane is $\frac{x}{a} + \frac{y}{a} + \frac{z}{b} = 1$

Partially differentiate with respect to $x$ and $y$ ,we get

$\frac{1}{a} + \frac{p}{b} = 0$

$p = -\frac{b}{a} \ldots (1)$

$\frac{1}{a} + \frac{q}{b} = 0$

$q = -\frac{b}{a} \ldots (2)$

From (1) and (2) we get

$p = q$

**10.Form a partial differential equation by eliminating the arbitrary constants a and b from the equation $(x - a)^2 - (y - b)^2 = z^2 cot^2 \alpha$**

Given $(x - a)^2 - (y - b)^2 = z^2 cot^2 \alpha \ldots (1)$

Partially differentiate with respect to $x$ and $y$ ,we get

$2(x - a) = 2zpcot^2\alpha$ □ $(x - a) = zpcot^2\alpha \ldots (2)$

$2(y - a) = 2zqcot^2\alpha$ □ $(y - b) = zqcot^2\alpha \ldots (3)$

Substituting (2) and (3) in (1) we get

$(zpcot^2\alpha)^2 + (zqcot^2\alpha)^2 = z^2cot^2\alpha$

$p^2cot^2\alpha + q^2cot^2\alpha = 1$

$p^2 + q^2 = tan^2\alpha$

**11.Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$?**

Given $(D^2 - 2DD' + D'^2)z = e^{x-y}$

P.I= $\frac{1}{D^2-2DD'+D'^2}e^{x-y}$

$= \frac{1}{1+2+1}e^{x-y}$ $[Replace\ D\ by\ 1\ and\ D'\ by - 1]$

$= \frac{1}{4}e^{x-y}$

**12.Solve the partial differential equation $pq = x$**

Given $pq = x$

$\frac{p}{x} = \frac{1}{q} = k$

$p = kx$ , $q = \frac{1}{k}$

$z = \int pdx + qdy$

$= \int kxdx + \int \frac{1}{k}dy$

$= k\left(\frac{x^2}{2}\right) + \frac{1}{k}y + c$

**13. Find the complete integral of $q = 2px$**

Given $q = 2px$

$xp = \frac{q}{2} = k$

$p = \frac{k}{x}$ $q = 2k$

$z = \int pdx + qdy$

$= k\int \frac{1}{x}dx + \int 2kdy$

$= klogx + 2ky + c$

**14.Form a p. d .e by eliminating the arbitrary constants from z=$ax^2 + ay^2 + b$**

Soln: p=$\frac{\partial z}{\partial x}$=$a^2$

q=$\frac{\partial z}{\partial y}$=2ay

$(2) \Longrightarrow y = \frac{q}{2a}$

$y^2 = \frac{q^2}{4a^2}$

$y^2 = \frac{q^2}{4a^2}$ by (1)

$4y^2 p = q^2$ which is the required p.d.e.

**15.Form the pde by eliminating a and b from $z = a(x + y) + b$**

Soln: Given $z = a(x + y) + b$

$p=\frac{\partial z}{\partial x}=a$ ......................(1)

$q=\frac{\partial z}{\partial y}=a$ ..........................(2)

From (1) and (2) we get the required p.d.e. ,

p=q.

**16. Form the general solution of $\frac{\partial^2 z}{\partial y^2}=0$**

Soln: Given $\frac{\partial^2 z}{\partial y^2}=0$

$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)=0$

Integrating p.w.r.to y on both sides

$\frac{\partial z}{\partial y}=f(x)$

Again integrating p.w.r.to y on both sides

$z = f(x)y + F(x)$ where both $f(x)$ and $F(x)$ are arbitrary.

**17.Obtain the complete solution of the equation $z = px + qy - 2\sqrt{pq}$**

Soln: Given $z = px + qy - 2\sqrt{pq}$

This is of the form $z = px + qy + f(p, q)$

Hence the complete integral is

$z = ax + by - 2\sqrt{ab}$ where a and b are arbitrary constants.

**18. Find the complete integral of $pq = xy$**

*Soln*: Given $pq = xy$

Hence $\frac{p}{x} = \frac{y}{q}$

It is of the form $f(x, p) = \varphi(y, q)$

Let $\frac{p}{x} = \frac{y}{q}$=a

□ $p = ax \ \ and\ q = \frac{y}{a}$

$Hence\ \ dz = pdx + qdy$

$dz = axdx + \frac{y}{q}dy$

$Integrating\ on\ b$oth sides

$z=a\frac{x^2}{2} + \frac{y^2}{2a} + c$

$2az = a^2x^2 + y^2 + b$ which is the required p.d.e.

**19. Solve $(D^2 + 6DD^1 + 9D^2)z = 0$**

Soln: The given pde is $(D^2 + 6DD^1 + 9D^2)z = 0$

The A.E is $m^2 + 6m + 9 = 0$

□ $(m + 3)(m + 3) = 0$

□ $m = -3, -3$

$The\ solution\ is\ z = f_1(y - 3x) + xf_2(y - 3x)$

**20.Find the particular integral of $(D^2 + 4DD^1)y = e^x$**

Soln; P.I $=\frac{1}{D^2+4DD^1}e^x$

$=\frac{1}{D^2+4DD^1}e^{x+0y}$

$=e^x\left(\frac{1}{1+4(1)(0)}\right)$

$=e^x$

**PART B**

1.Find the general solution of $x(y - z)p + y(z - x)q - z(x - y) = 0$

2. Find the singular solution of $z = px + qy + p^2 - q^2$

3. solve $(D^2 + 2DD' - 6D'^2)z = -x\sin y$

4. Solve $(D^2 + 2DD' - 6D'^2)z = \cos(x + 2y)$

5.Solve the partial differential equation given by $(x^2-y^2-z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$

6.Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y - 1)e^x$

7.Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$

8. Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = e^{2x-y} + 3$